1. 1. Base case:

and

Inductive step:

Assume

Prove

both are , therefore, and

* 1. Base case: 4 is proven by inductive definition(1).

Therefore, 4 is true too.

Inductive Step:

Assume x and y , and x and y are

Prove xy

So xy . Therefore

1. (I1) if x ∈ BP then {x} ∈ BP , and (I2) if x ∈ BP and y ∈ BP then xy ∈ BP .

X = {x}

If x ∈ BP, which means {x} ∈ BP, By (I1).

{x} has same amount of { and } which is one for each.

Therefore, we can say that if x ∈ BP, then x has the same number of { and }.

Base case: T has only a single vertex. So, there’s only 1 root and no vertex. Therefore, it satisfies

Recursive step:

Let T1 and T2 be left root and right root.

So and

Vertex

Leaves

* 1. Base case: (0, 0)

Inductive definition: if (a, b), then , and

* 1. Base case: (a, b) = (0, 0)

which

if (a, b), then , and

Since (a, b) and a – b is multiple of 4. So,

All sets we added to L’ will always be in L, therefore,

* 1. Let (s, t) be an arbitrary element of L, which means (s, t) where x – y is multiple of 4. t can be written as s – 4k for k any positive integer.

Case 1: s and k are positive. If move for s times and for k times, then we will get .

Case 2: s and k are negative. . If move for -s times and for -k times, then we will get .

Case 3: s is positive, and k is negative. If move for s times and for -k times, then we will get .

Case 4: s is negative, and k is positive If move for -s times and for k times, then we will get .

Therefore, we can say that

* 1. No, I think my inductive definition is not unambiguous. There is other unique way to produce x using my rules.